Math 70 12.6–3rd Log + Exp Equations Lesson 43

Mixed practice solving log equations and exponential equations.

Exam 4 topics list

Lesson 43

Math 70 EXAM #4 Topics List

Math 70-33: Exam #4 Monday, 30 April, 1:20 PM – 2:25 PM
Math 70-34: Exam #4 Thursday, 26 April, 1:20 PM – 2:25 PM

Optional Exam 4 Review session / office hours for any Math 70 student in either section:
Wednesday, April 25, 3:30-4:30 PM, ASC room 420.

Martin-Gay		**Bittinger**
Lecture sections	**Lesson number**	**Lecture sections**
9.1	33	12.1-1st
9.2-2nd	34	12.1-2nd
	35	(previous exam)
9.3	36	12.2
9.5	37	12.3-1st
9.5 & 9.6	38	12.3-2nd & 12.4-1st
9.6 & 9.7	39	12.4-2nd*
9.7	40	12.5
9.7 & 9.8	41	12.6-1st
9.8	42	12.6-2nd
9.8	43	12.6-3rd
	44	(material on next exam)
	45	(material on next exam)
	46	(material on next exam)
	47	EXAM 4 (c12)

If you are repeating Math 70 or study with students in other sections of Math 70, it may be helpful to know how the other textbook, Martin-Gay, aligns with your textbook, Bittinger.

(1) Grandma invested \$20,000 in a stock fund earning 8% interest compounded continuously. When will she have \$50,000 in the account? Round to the nearest tenth of a year.

"compounded continuously" ⇒ key words!

Continuously Compounded Interest Formula
$A = Pe^{rt}$

"pert"

* Memorize!

$A = 50000$
$P = 20000$
$r = .08$
$t = ?$

Substitute $\quad 50{,}000 = 20{,}000\, e^{.08t}$

isolate exponential by dividing by 20,000

$$\frac{50{,}000}{20{,}000} = e^{.08t}$$

$$2.5 = e^{.08t}$$

exponential is on the RHS!

$$e^{.08t} = 2.5$$

Method 1: equivalent log equation

$$\log_e 2.5 = .08t$$

$$\frac{\ln 2.5}{.08} = \frac{.08t}{.08}$$

$$t = \frac{\ln(2.5)}{.08}$$

Method 2: take ln both sides

$$\ln e^{.08t} = \ln 2.5$$

inverse property!

$$.08t = \ln 2.5$$

$$t = \frac{\ln(2.5)}{.08}$$

$t \approx 11.45$

$\boxed{t \approx 11.5 \text{ yrs}}$

Math 70 Mixed Practice 9.8

Solve for exact answers, then find approximations with four decimal places.

1) $\ln(3x)-\ln(x-3)=2$

6) $5^{3x-5}=4$

2) $3^{2x}=7$

7) $\log_2 x+\log_2 2x-3=1$

3) $\log_5 2+\log_5 x=2$

8) $3\cdot 4^{x+5}=2$

4) $3^{2x+1}=6$

9) $-\log_6(4x+7)+\log_6 x=1$

5) $\log(5x)-\log(x+1)=4$

10) $2\cdot 5^{x-1}=1$

Math 70 Mixed Practice

Key

Solve for exact answers, then find approximations with four decimal places.

$\log_5 4 = 3x-5$ $\quad \frac{\log_5(4)+5}{3} = x$

1) $\ln(3x) - \ln(x-3) = 2$

$\ln\left(\frac{3x}{x-3}\right) = 2$

$e^2 = \frac{3x}{x-3}$

$e^2x - 3e^2 = 3x$

$-3e^2 = 3x - e^2x$

$-3e^2 = x(3-e^2)$

$\boxed{x = \frac{-3e^2}{(3-e^2)}}$

$x \approx 5.05055$

$\boxed{5.0506}$

2) $3^{2x} = 7$

$\log 3^{2x} = \log 7$

$2x \log 3 = \log 7$

$x = \boxed{\frac{\log 7}{2\log 3}} = \boxed{\frac{\log 7}{\log 9}} \approx 0.88562$

$\approx \boxed{.8856}$

3) $\log_5 2 + \log_5 x = 2$

$\log_5 2x = 2$

$5^2 = 2x$

$25 = 2x$

$\frac{25}{2} = x$

$x = \boxed{12.5}$

4) $3^{2x+1} = 6$

$\log 3^{2x+1} = \log 6$

$(2x+1)\cdot\log 3 = \log 6$

$2x+1 = \frac{\log 6}{\log 3}$

$2x = \frac{\log 6}{\log 3} - 1$

$\boxed{x = \frac{\log 6}{2\log 3} - \frac{1}{2}}$

$x = .31546$

$x \approx \boxed{.3155}$

5) $\log(5x) - \log(x+1) = 4$

$\log \frac{5x}{x+1} = 4$

$10^4 = \frac{5x}{x+1}$

$10000(x+1) = 5x$

$10000x + 10000 = 5x$

$9995x = -10{,}000$

$x = \frac{-10{,}000}{9995} = \cancel{\frac{-2000}{1999}}$

$x \approx -1.00050$

$\boxed{x \approx -1.0005}$

$\boxed{\text{NO SOLUTION}}$

6) $5^{3x-5} = 4$

$\log 5^{3x-5} = \log 4$

$(3x-5)\cdot\log 5 = \log 4$

$3x - 5 = \frac{\log 4}{\log 5}$

$3x = \frac{\log 4}{\log 5} + 5$

$x = \frac{\log 4}{3\log 5} + \frac{5}{3}$

$\boxed{x = \frac{\log 4}{\log 125} + \frac{5}{3}}$

$x \approx 1.95378$

$\boxed{x \approx 1.9538}$

7) $\log_2 x + \log_2 2x - 3 = 1$

$\log_2(x\cdot 2x) = 4$

$\log_2 2x^2 = 4$

$2^4 = 2x^2$

$16 = 2x^2$

$8 = x^2$

$x = +\sqrt{8}$ or $\cancel{-\sqrt{8}}$

$x = \sqrt{8} =$

$\boxed{x = 2\sqrt{2}}$

$x = 2.82842$

$\boxed{x \approx 2.8284}$

8) $\frac{3\cdot 4^{x+5}}{3} = \frac{2}{3}$

$4^{x+5} = \frac{2}{3}$

$(x+5)\log 4 = \log 2/3$

$x+5 = \frac{\log 2/3}{\log 4}$

$\boxed{x = \frac{\log 2/3}{\log 4} - 5}$

$x = -5.29248$

$\boxed{x \approx -5.2925}$

9) $-\log_6(4x+7) + \log_6 x = 1$

$\log_6 x - \log_6(4x+7) = 1$

$\log_6\left(\frac{x}{4x+7}\right) = 1$

$6 = \frac{x}{4x+7}$

$6(4x+7) = x$

$24x + 42 = x$

$23x = -42$

$\cancel{x = \frac{-42}{23}}$

$\boxed{\text{NO SOLUTION}}$

10) $\frac{2\cdot 5^{x-1}}{2} = \frac{1}{2}$

$5^{x-1} = \frac{1}{2}$

$(x-1)\log 5 = \log\left(\frac{1}{2}\right)$

$x - 1 = \frac{\log(1/2)}{\log 5}$

$\boxed{x = \frac{\log .5}{\log 5} + 1}$

$x = .56932$

$\boxed{x \approx .5693}$

☆ Absolutely do not ever try to "divide by" a log.

$\log_2 x = 7$ means $2^7 = x$

$128 = x$

NOT ~~$\log_2 x = 7$~~ ~~$x = \frac{7}{\log_2}$~~

This says you have learned very little, so it will get zero credit.

Ditto $\ln x = 2$ means $e^2 = x$

NOT ~~$x = \frac{2}{\ln}$~~

☆ ln or log without an argument is nonsense. We must always take the log or ln <u>of</u> something. $\ln(x)$ $\log(2)$

Our main strategies for equations:

If we have a log, write an exponential.

If we have an exponential, take logs of both sides of the eqn.

9.8

9.7.47

Use the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ to solve the compound interest problem.

Find how long it takes for $1500 to double if it is invested at 4% interest compounded monthly.

The money will double in value in approximately $\boxed{8.7}$ years.
(Do not round until the final answer. Then round to the nearest tenth as needed.)

$P = 1500$

$A =$ double $1500 = 3000$

$r = .04$

$n = 12$ (monthly ⇒ 12 times per year)

$t =$ unknown.

$$\underbrace{3000 = 1500}_{\text{div by 1500 to isolate the exponential.}}\underbrace{\left(1+\frac{.04}{12}\right)^{12t}}_{\text{exponential}}$$

$$\frac{3000}{1500} = \left(1+\frac{.04}{12}\right)^{12t}$$

Simplify

$$2 = \left(1+\frac{.04}{12}\right)^{12t}$$

Notice: Double reduces to 2. You can start at this step if you remember this.

Take logs

$$\log 2 = \log\left(1+\frac{.04}{12}\right)^{12t}$$

Log property #3:

$$\log 2 = 12t \cdot \log\left(1+\frac{.04}{12}\right)$$

$1+\frac{.04}{12} = \frac{301}{300}$ if you prefer

Isolate t: by dividing

$$\frac{\log 2}{12\log\left(\frac{301}{300}\right)} = t$$

Calculate log(2)/(12* log(301/300)) enter

$t \approx 17.35 \Rightarrow \boxed{t \approx 17.4 \text{ yrs}}$

China is experiencing an annual growth rate of 0.606 % In 2007, the population of China was 1,321,851,888. How long will it take for the population to be 1,500,000,000? Round to the nearest tenth of a year.

$y = y_0 e^{rt}$ population model

y = population at time t = 1,500,000,000

y_0 = population at time 0 = 1,321,851,888

r = growth rate = 0.606% = 0.00606

$1\,500\,000\,000 = 1,321,851,888\, e^{0.00606t}$ subst given values.

$\dfrac{1\,500\,000\,000}{1\,321\,851\,888} = e^{0.00606t}$ isolate exponential

$\ln\left(\dfrac{1\,500\,000\,000}{1\,321\,851\,888}\right) = \ln e^{0.00606t}$

$\ln x = \log_e x$
so RHS is $\log_e e^{\text{stuff}} = \text{stuff}$

$\ln\left(\dfrac{1\,500\,000\,000}{1\,321\,851\,888}\right) = 0.00606t$ isolate t

$\dfrac{\ln\left(\dfrac{1\,500\,000\,000}{1\,321\,851\,888}\right)}{0.00606} = t$

$t \simeq 20.86\ldots$ $t=0$ means 2007

$\boxed{20.9 \text{ yrs}}$ $t = 20 \Rightarrow$ means 2027

during 2027 the population will pass 1.5 billion

9.7.49 Use the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ to solve the compound interest problem.

Find how long it takes a $1400 investment to earn $500 interest if it is invested at 2% interest compounded quarterly.

$500 interest will be earned in approximately ☐ years. (Round to the nearest tenth.)

$P = 1400$

$A = 1400 + 500 = 1900$ ← "earn $500 interest" means add 500 to account

$r = .02$

$n = 4$ (quarterly ⟹ 4 times per year)

t = unknown

$$1900 = 1400\left(1 + \frac{.02}{4}\right)^{4t}$$

Proceed as in previous problem
- isolate exponential
- take logs
- use log property #3
- isolate variable
- calculate & round.

$$\frac{1900}{1400} = (1.005)^{4t} \quad \text{or} \quad \left(\frac{201}{200}\right)^{4t}$$ - isolate exponential, - simplify the base of exponential

$$\frac{19}{14} = (1.005)^{4t} \quad \text{or} \quad \frac{19}{14} = \left(\frac{201}{200}\right)^{4t}$$

$$\log\left(\frac{19}{14}\right) = \log(1.005)^{4t} \quad \text{or} \quad \log\left(\frac{19}{14}\right) = \log\left(\frac{201}{200}\right)^{4t}$$ - take logs both sides

$$\log\left(\frac{19}{14}\right) = 4 \cdot t \cdot \log(1.005) \quad \text{or} \quad \log\left(\frac{19}{14}\right) = 4 \cdot t \cdot \log\left(\frac{201}{200}\right)$$ - log property

$$\frac{\log\left(\frac{19}{14}\right)}{[4\log(1.005)]} = t \quad \text{or} \quad \frac{\log\left(\frac{19}{14}\right)}{\left[4\log\left(\frac{201}{200}\right)\right]} = t$$ - isolate t

$15.307\ldots \approx t$

$\boxed{15.3 \text{ years} \approx t}$

9.7.51 **Find how long it takes \$2000 to double if it is invested at 5% interest compounded semiannually.**

Use the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ **to solve the compound interest problem.**

It will take approximately ☐ years.

(Do not round until the final answer. Then round to the nearest tenth as needed.)

$P = 2000$

$A = 2(2000) = 4000$

$r = .05$

$n = 2$ (semi-annually means 2 times per year)

$t =$ unknown

$$4000 = 2000\left(1 + \frac{.05}{2}\right)^{2t}$$

proceed as in previous problem

- isolate exponential
- take logs
- use log property # 3
- isolate variable
- calculate & round.

$$\frac{4000}{2000} = \left(\frac{41}{40}\right)^{2t} \quad \text{or} \quad \frac{4000}{2000} = (1.025)^{2t}$$

- isolate exponential
- simplify the base of exponential.

$$2 = \left(\frac{41}{40}\right)^{2t} \quad \text{or} \quad 2 = (1.025)^{2t}$$

$$\log(2) = \log\left(\frac{41}{40}\right)^{2t} \quad \text{or} \quad \log(2) = \log(1.025)^{2t}$$

$$\log(2) = 2t \cdot \log\left(\frac{41}{40}\right) \quad \text{or} \quad \log(2) = 2 \cdot t \cdot \log(1.025)$$

$$\frac{\log(2)}{\left[2\log\left(\frac{41}{40}\right)\right]} = t \qquad \frac{\log(2)}{2\log(1.025)} = t$$

$14.0355 \approx t$

$\boxed{14.0 \text{ years} \approx t}$